



Assignment (2)

1. Obtain the transfer function and draw block diagram of the differentiating circuit shown in Figure (1).

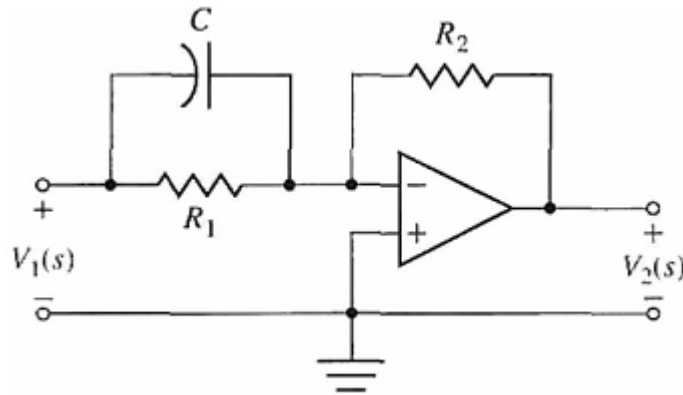


Fig. 1. A differentiating circuit

2. Figure (2) shows two pendulums suspended from frictionless pivots and connected at their midpoints by a spring. Assume that each pendulum can be represented by a mass M at the end of a massless bar of length L . Also assume that the displacement is small and linear approximations can be used for $\sin \theta$ and $\cos \theta$. The spring located in the middle of the bars is unstretched when $\theta_1 = \theta_2$. The input force is represented by $f(t)$, which influences the left-hand bar only, (a) Obtain the equations of motion, and sketch a block diagram for them, (b) Determine the transfer function $T(s) = \theta_1(s)/F(s)$.

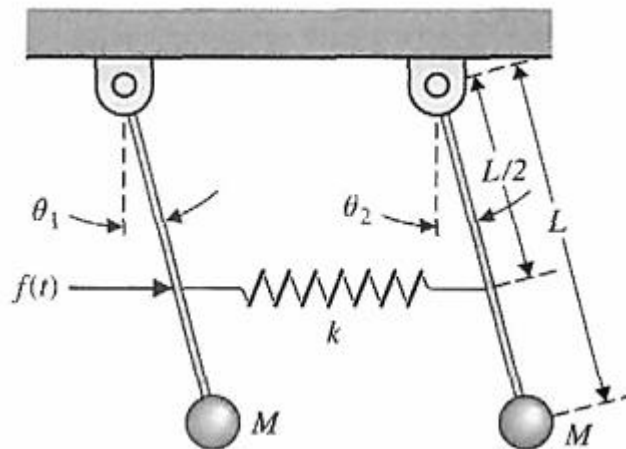


Fig. 2. The bars are each of length L and the spring is located at $L/2$



3. An electromechanical open-loop control system is shown in Figure (3). The generator, driven at a constant speed, provides the field voltage for the motor. The motor has an inertia J_m and bearing friction b_m . Obtain the transfer function $\theta_L(s)/V_f(s)$ and draw a block diagram of the system. The generator voltage v_g can be assumed to be proportional to the field current i_f .

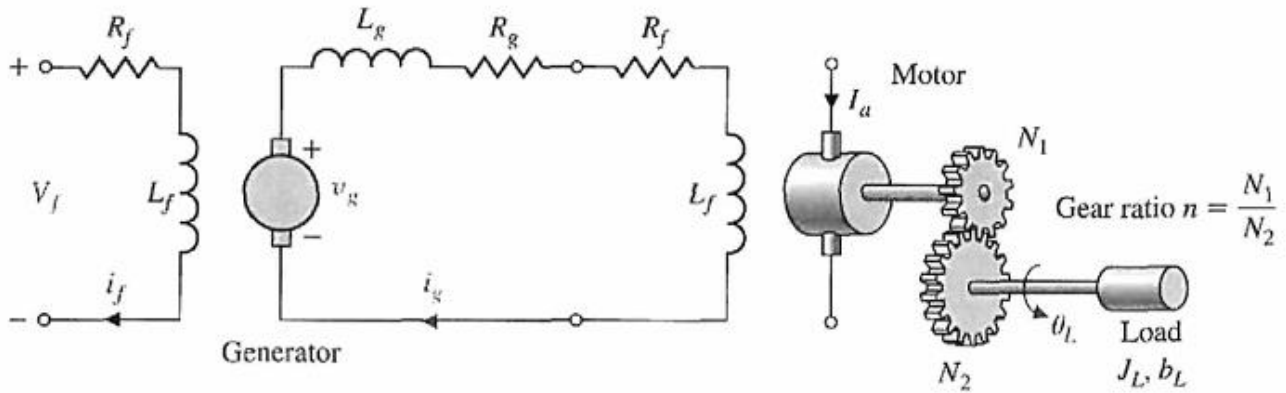


Fig. 3. Motor and generator

4. The circuit shown in Figure (4) is called a lead lag filter.
- Find the transfer function $V_2(s)/V_1(s)$. Assume an ideal op-amp.
 - Determine $V_2(s)/V_1(s)$ when $R_1 = 100\Omega$, $R_2 = 200\Omega$, $C_1 = 1\mu F$, and $C_2 = 0.1\mu F$.
 - Determine the partial fraction expansion for $V_2(s)/V_1(s)$.

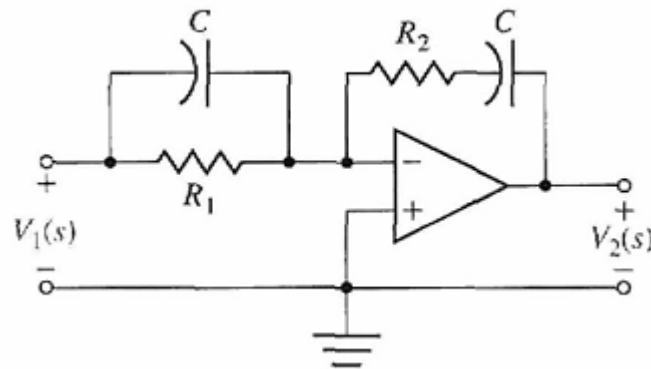


Fig. 4. Lead-lag filter



5. Derive the transfer functions for the operational amplifier circuits shown in Figure (5).

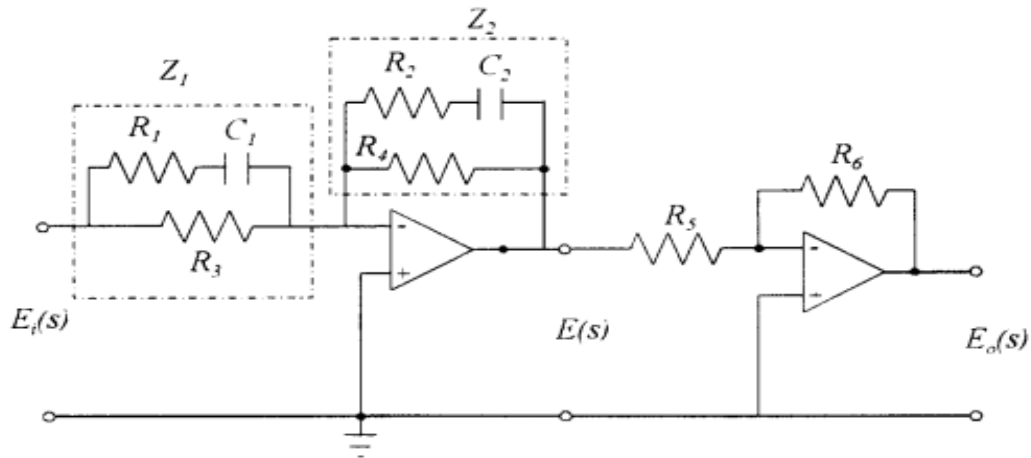


Fig. 5. Operational amplifier

6. Determine a state-space representation for the system shown in Figure (6). The motor inductance is negligible, the motor constant is $K_m = 10$, the back electromagnetic force constant is $K_b = 0.0706$, the motor friction is negligible. The motor and valve inertia is $J = 0.006$, and the area of the tank is 50 m^2 . Note that the motor is controlled by the armature current i_a . Let $x_1 = h$, $x_2 = \theta$, and $x_3 = d\theta/dt$. Assume that $q_1 = 80\theta$, where θ is the shaft angle. The output flow is $q_o = 50h(t)$.

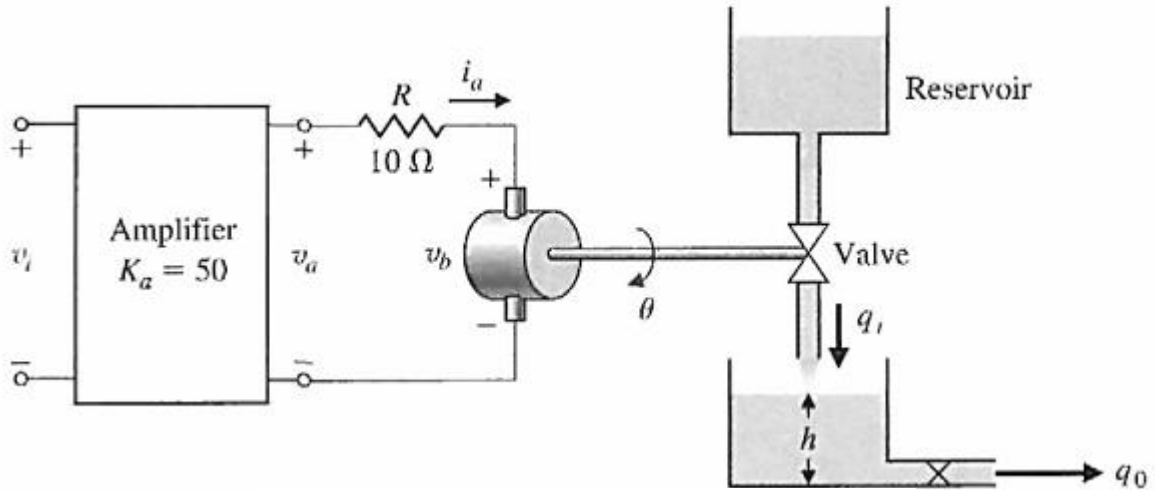
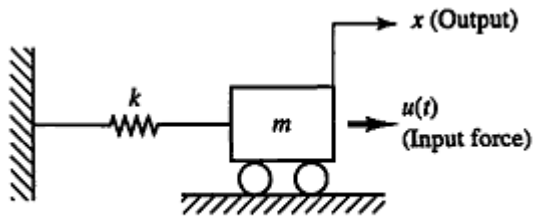
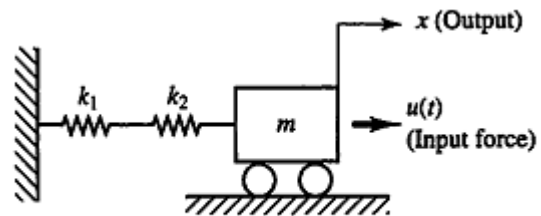


Fig. 6. One-tank system

7. Obtain state-space representations of the mechanical systems shown in Figures (7) a and b.



(a)



(b)



8. Obtain a state-space representation of the mechanical system shown in Figure (7). The force $u(t)$ applied to mass m_1 is the input to the system. The displacements y and z are the outputs of the system. Assume that y and z are measured from their respective equilibrium positions.

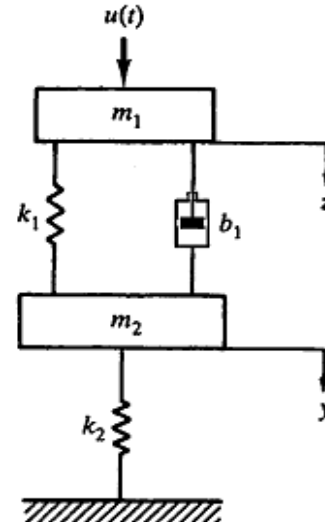


Fig. 8. Mechanical system

9. Given the state equation

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & -3 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u$$

And output equation

$$y = [1 \ 0 \ 0] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Obtain the corresponding scalar differential equation in terms of y and u .

10. Obtain the unit-step response curve and unit-impulse response curve of the following system with MATLAB.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -5 & -25 & -5 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u$$

$$y = [0 \ 25 \ 5] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + [0]u$$